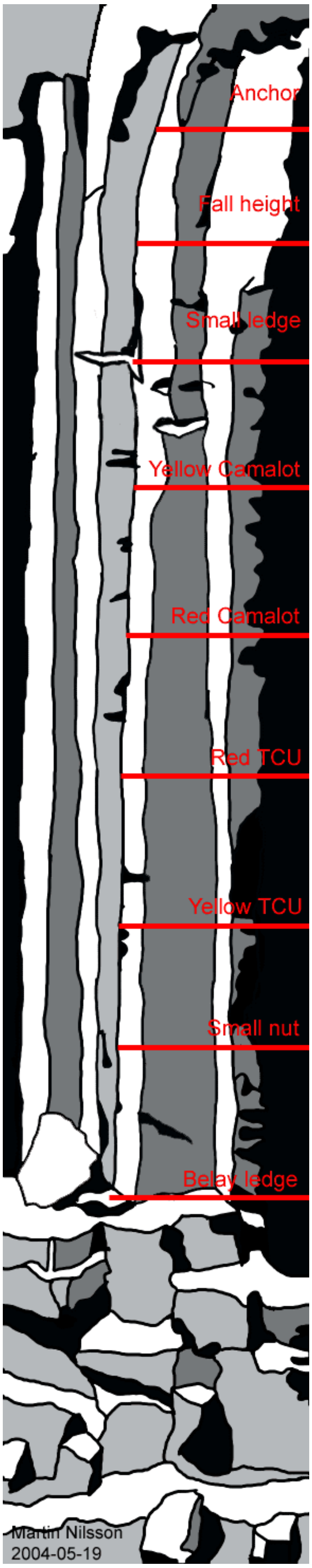


Analysis of the accident on Air Guitar

The Safety Committee of the Swedish Climbing Association

Draft 2004-05-30

Svenska
KLÄTTERFÖRBUNDET

Preface

The Swedish Climbing Association (SKF) Safety Committee's overall purpose is to reduce the number of incidents and accidents in connection to climbing and associated activities, as well as to increase and spread the knowledge of related risks.

The fatal accident on the route Air Guitar involved four failed pieces of protection and two experienced climbers. Such unusual circumstances ring a warning bell, calling for an especially careful investigation.

The Safety Committee asked the American Alpine Club to perform a preliminary investigation, which was financed by a company formerly owned by one of the climbers. Using the report from the preliminary investigation together with additional material, the Safety Committee has analyzed the accident. The details and results of the analysis are published in this report.

There is a large amount of relevant material, and it is impossible to include all of it in this report. The Safety Committee has been forced to select what has been judged to be the most relevant material. Additionally, the remoteness of the accident site, and the difficulty of analyzing the equipment have complicated the analysis. The causes of the accident can never be "proven" with certainty. This report is not the final word on the accident, and the conclusions may need to be changed if new information appears. However, we do believe we have been able to gather sufficient evidence in order to attempt an explanation of the accident in this report.

The Safety Committee

Stockholm, 2004-05-21

Martin Nilsson, D.Eng, SKF Safety Committee member, Head of investigation
Jan Leyon, M.D., SKF Safety Committee chairman
Mårten Johansson, SKF Safety Committee member
Magnus Nilsson, Swedish member of the UIAA Safety Commission

Analysis of the accident on Air Guitar

The Safety Committee, Swedish Climbing Association
2004-05-21

Summary: In this report, we describe and analyze a fatal rock climbing accident near Vantage, Washington, USA, on September 30, 2002. We first overview rock climbing and its belay system, before describing the accident. We summarize an earlier investigation, and collect some additional data for our analysis. After a theoretical analysis and computer simulations, we deduce a combination of factors that most likely caused the accident together. We conclude the report by discussing how similar accidents could be avoided in the future.

The purpose of this report is to help preventing similar accidents. The intention is not to point out any person as “guilty”. In fact, the conclusion of the report shows that responsibility for the accident does not lie on any single person.

1. Introduction

1.1 Structure of this report

The remainder of section 1 *Introduction* introduces the key safety concepts of rock climbing. This section, section 2 *Accident description*, and section 7 *Conclusions* are written in order to be understandable without extensive rock climbing experience or knowledge of physics.

Section 3 summarizes the preliminary investigation. Data from the preliminary investigation is used extensively throughout this report. Section 4 *Additional information* presents some material not available in the preliminary investigation report. These sections require firm knowledge and experience of rock climbing.

Sections 5 *Analysis* and 6 *Discussion* analyze the accident using physics principles and computer simulations. These sections can be understood without a strong physics background, but the theory behind the analysis is somewhat demanding and is included in Appendices A: *The Physics of a Fall* and B: *Computer simulation*.

1.2 The belay system

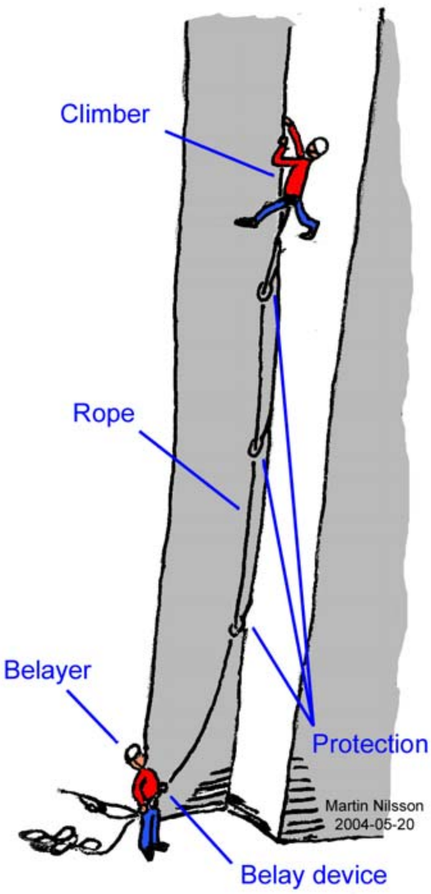
The safety of rock climbing depends on the *belay system* (Fig 1-1). A climber ties a *climbing rope* to the *harness* before starting to climb. This rope will catch the climber in case of a fall. As the climber climbs up a rock, the climber places *protection* into cracks along the route on the rock. The protection catches the climber in case of a fall.

There are many types of protection, but they share the property that they can hold a force directed downwards. One common type of protection are *nuts*, which are small wedge-shaped pieces of metal (Photo 1-1). When a nut is properly wedged into a crack, it is able to hold a heavy load, such as that of a falling climber. Another type of protection is *camming devices* (Photo 1-2). These devices are more complex, but can be easily placed also in wide and/or expanding cracks. They consist of lever mechanisms, which expand when loaded.

After placing protection, the climber doesn't tie the rope directly to the protection, but instead hooks the rope into a *carabiner*, which in its turn is hooked into the protection, usually with an intermediate *sling* (Photo 1-3). In this way, the rope can slide through the protection when the climber climbs on. The rope runs from the climber, through the carabiners attached to the protection, and down to the *belayer*. The belayer is normally attached to the rock, in order not to be tossed up in the air by a falling climber.

As the climber proceeds, the belayer feeds out rope to the climber, but on a fall the rope must be stopped. In order to accomplish this, the belayer uses a *belay device*. A common type of belay device has an oblong hole, through which a bight of the rope is fed (Photo 1-4). Clipping a carabiner through the bight generates friction and enables the belayer to arrest a fall with his hand on the rope. Additionally, the belay device should work as an “emergency valve”, by limiting the maximum rope load. This is discussed further below.

The equipment shown in these pictures is similar to, but not the same as that used in the accident.



Figur 1-1: The belay system.

1.2 The three links of the belay chain

The *protection*, the *belay device*, and the *rope* together constitute the core of the belay chain. They are all directly involved in controlling and absorbing the large forces involved in a fall. The belay system is designed to work even if one link malfunctions, but if several links fail, the system will not work.

Photo 1-1: Nuts.

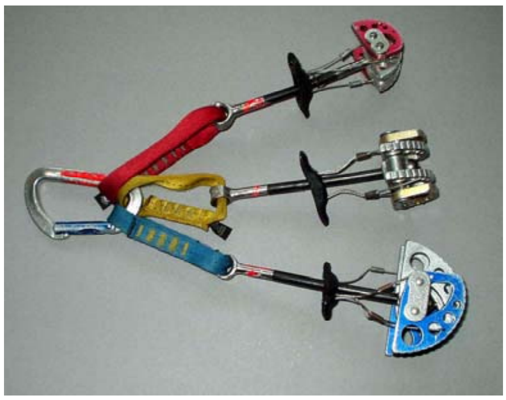


Photo 1-2: Camming devices (Camalots).

Photo 1-3: Sling and carabiner holding a rope. **Photo 1-4:** Belay device (Reverso).

Assume that a climber falls. The load on the topmost protection is the sum of two components. One is the force F_x in the section of the rope from the topmost protection down to the falling climber. The other is the force F_y in the rope going down to the belayer (Fig. 1-2). Due to friction in the carabiner, F_y is typically about 70% of F_x . The total load on the protection is $F = F_x + F_y = 1.7\ F_x = 2.4\ F_y$. Well-placed protection can accept heavy loads, but not arbitrarily large. The breaking strength of good protection is limited to, typically, around 10-15 kN. As a comparison, the breaking strength of ropes is well over 20 kN [12]. An additional consideration is that the load on the climber (F_x) must be limited. Forces above 6 kN on the climber will be felt as quite heavy, and at 12 kN, the climber risks serious injury [10]. A conclusion is that the belay chain should limit F_x to around 6-9 kN, and consequently F_y to 4.5-6.5 kN. The UIAA norm specifies that the shock load on an 80-kg climber must be less than 12 kN for a fall factor 1.77 fall with a new rope [13], but for modern ropes, the load is usually specified to be around 7-8 kN. This corresponds to a load on protection of about 11.9-13.6 kN.

1.2.1 The protection

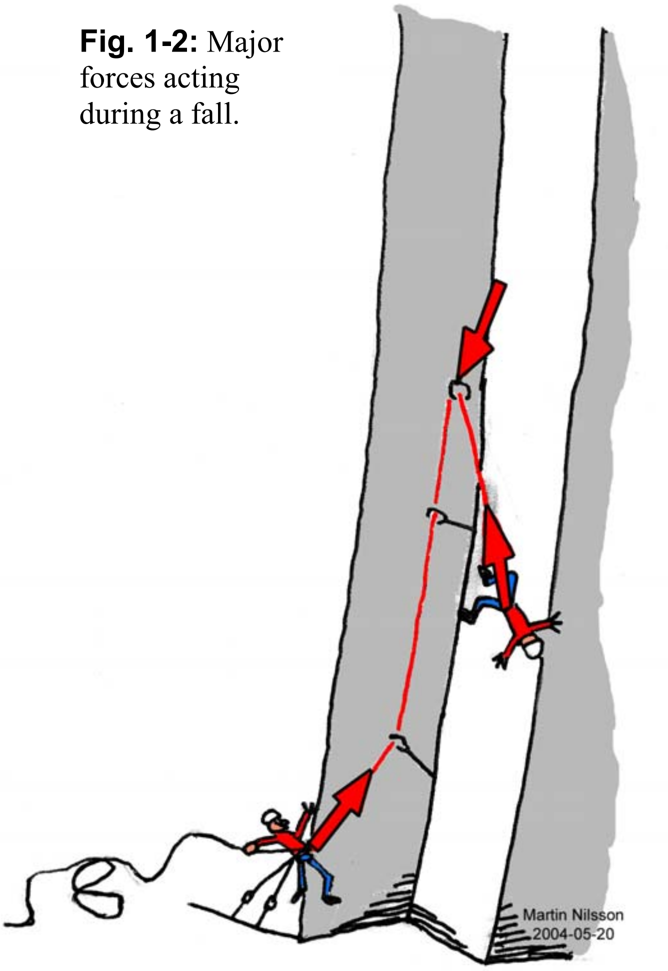
Protection, which is badly set, may not hold much load at all. Fortunately, since multiple points of protection are placed, if the first piece of protection fails, there are others that will be able to hold the load. It is important to note that multiple pieces of protection protect against bad placement, but not against excessive load. In the terminology of probability theory, bad placements are approximately independent, but excessive load is not.

The fall factor f is the fall distance d, divided by the length of rope L between the climber and the belayer.

$$F = d/L$$

Typically, f is less than twice the distance between the pieces of protection, divided by the climbed distance. f is obviously always less than two. The maximum force in the rope depends on the fall factor, rather than on the fall distance. A more precise calculation of the maximum load can be done, taking friction and other factors into account [Appendix A], but the conclusion that the force depends on the fall factor rather than the fall distance still holds. The climber can reduce the fall factor by spacing the pieces of protection more closely. This is most important at the start of the climb, where even a short fall can lead to a large fall factor.

Fig. 1-2: Major forces acting during a fall.



1.2.2 The rope

The elasticity of the rope is key to limiting the forces in the belay chain. The elasticity is expressed in terms of *static elongation* for small loads, i.e. how much the rope strains (extends) for a static load of 80 kg. Ordinary, *"dynamic"* climbing ropes have a static elongation approximately in the range 6-8%, i.e. they become 6-8% longer for a static load of 80 kg. Ropes with static elongation under 3% are *"static"* and are unsuitable for lead climbing, because of the large forces generated during falls.

Ropes that are originally dynamic can become static due to age or handling. Frequent top roping or rappelling will reduce the elasticity of a rope [2].

1.2.3 The belay device

Primarily, the belay device is used for feeding out rope slowly, but stopping the feed immediately on a fall. An important secondary function of the belay device is to allow the rope to slip through if rope tension is too large, preventing overloading the protection, also known as *dynamic braking*. Belay devices are usually more or less dynamic; allowing the rope to slip through for loads F_y in the range 2-3kN, except for "static" type brakes, which may have twice this amount [3]. The limit depends on several factors, including the properties of the rope and the belayer's handling.

Consequently, a properly used belay device limits the force on the protection to 2.4 Fy = 2.4-7.2 kN, and on the climber to $F_x = F_y/0.7$ = 1.5-4.5 kN, depending on the type of device.

1.2.4 Introduction to fall physics*

In this subsection, we introduce the physics of a fall, and the principles behind estimating the rope elasticity from fall data. For a more careful and detailed analysis, please refer to appendix A. Readers uninterested in the physics can skip this section.

In this introduction, we assume that the friction between rope and carabiners can be ignored, for the sake of simplicity. Generalization to non-zero friction is not difficult, but lengthier. A climber with mass M falls without slack from height L above the belayer, having static belay at ground level. The potential energy E_{pot} , lost in the fall, is converted to energy stored in the rope, $E_{rope} \leq E_{pot}$. If we ignore friction, the load on the topmost piece of protection is twice the rope tension, $F = 2F_{rope}$. We assume that the rope is linearly elastic, i.e. it satisfies the equation $F_{rope} = k \cdot x$, where x is the absolute elongation of the rope, and k is the rope's spring constant. If the rope is linearly elastic, k can be calculated from the rope's static elongation ε_{80} by

$$k = \frac{80g}{L\varepsilon_{80}},$$

where $g = 9.80665$ m/s^2. The energy stored in the rope is the same as for linear springs,

$$E_{rope} = \frac{kx^2}{2} = \frac{(kx)^2}{2k} = \frac{(F_{rope})^2}{2k} = \frac{(F/2)^2}{2k} = \frac{F^2}{8} \cdot \frac{1}{k} = \frac{F^2}{8} \cdot \frac{L\varepsilon_{80}}{80g} \leq E_{pot} = Mg \cdot L,$$

from which we can derive $\varepsilon_{80} \leq 640 Mg^2 / F^2$. For $F = 100$ kg and $F \geq 15$ kN we have

$$\varepsilon_{80} \leq 640 \cdot 100 \cdot 9.80665^2 / 15000^2 < 0.0274 .$$

With the assumptions above, in order for the load on protection to reach 15 kN before reaching ground level, the rope can have a static elongation of at most 2.8%. Suppose instead that the static elongation is 6%. Then,

$$F^2 \leq 640 Mg^2 / \varepsilon_{80} < 103 \times 10^6 \text{N}^2,$$

so the load on the protection is at most the square root of this value, ie. $F < 10.2$ kN. If the static elongation is 6%, then the load on any piece of protection is less than 10.2 kN, even with static belay.

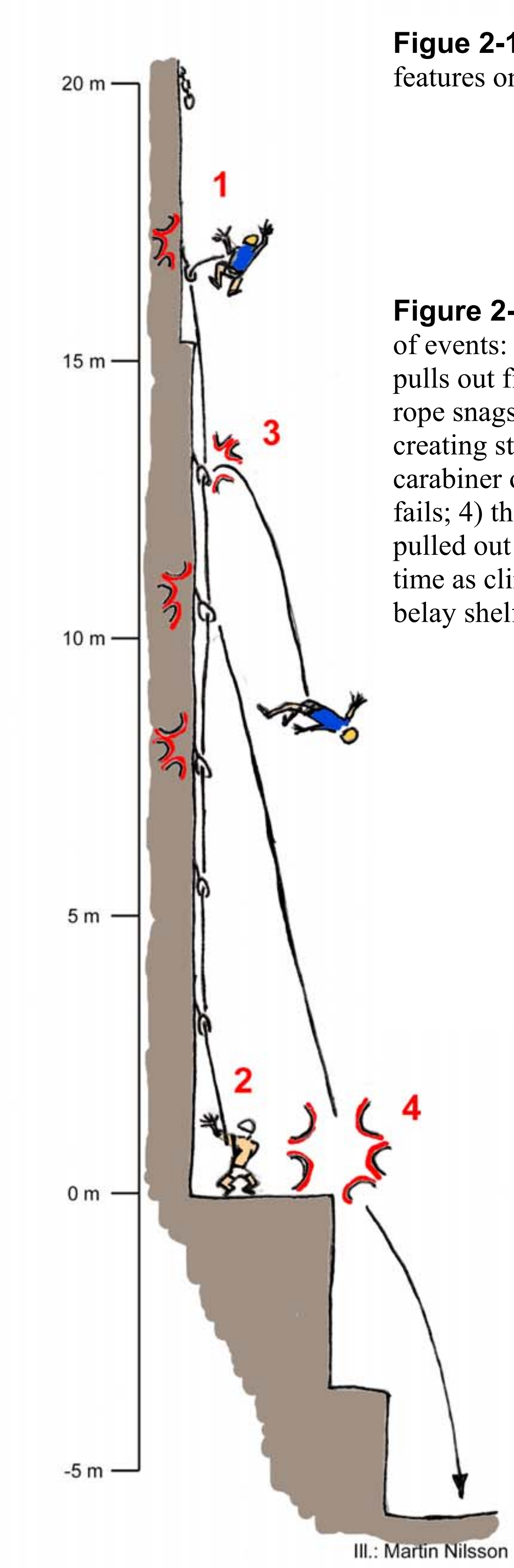
2. Accident description

The accident happened on a route called Air Guitar, in the crag area Frenchman Coulee, near Vantage, Washington, USA. The route is a clean, vertical crack climb without significant protrusions (Fig. 2-1, 2-2). The route is naturally protected, i.e. the climber needs to place the protection. It is of moderate difficulty, rated as 5.10a on the American scale, corresponding to approximately 6-, Swedish scale, or 6a, French scale. The 100-kg climber fell at the crux, near the top of the route. During the fall, the first piece of protection, a Black Diamond #3 Camalot, was pulled out. After this, the rope snagged the 90-kg belayer's arm, effectively causing static belay. At the next piece of protection, the carabiner holding the rope snapped. The third and fourth pieces of protection, both camming devices (Black Diamond #1 Camalot and Metolius #3 TCU) also pulled out, the climber’s head struck the belay ledge, and the climber finally came to rest on the footpath, below the belay ledge. The shock load on the belayer's arm caused damage, but due to the short duration, it didn't lift the belayer significantly. The belayer was unanchored.

The heights of the different points of interest are given in, or can be computed from the preliminary report and additional photographs (Table 2-1). The height of the #2 yellow TCU was measured from a combination of photographs [4], [14]. The heights for the remaining three pieces (#1 Camalot, #3 TCU, and a small nut) have been estimated by dividing the space uniformly between the known positions.

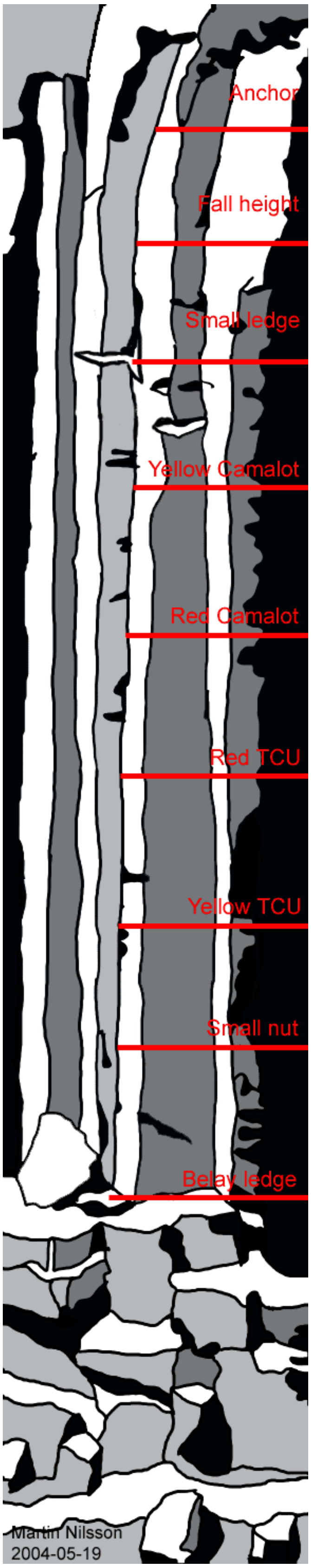
Feature	Height [feet]	Height [m]	Strength [kN]	Comments
Top of route	65	19.5		
Climber (knot)	58	17.4		Position used for drop tests
Climber's feet	55-56	16.5-16.8		
1. #3 Blue Camalot	53-57	15.9-17.1	11	Pulled out (prob. bad set)
Small ledge	51	15.3		
Drop test on #2 Camalot	43	12.9	15	Pulled out, well set
2. #2 Yellow Camalot	41-46	12.3-13.8	8.67?	Carabiner broken (prob. open)
3. #1 Red Camalot	(35)	(10.5)	15	Pulled out, cam damage
4. #3 Red TCU	(26)	(7.8)	13.3	Pulled out, bent axle
5. #2 Yellow TCU	17	5.1		Only protection left after fall
6. Small nut	(8)	(2.4)		Lifted out by rope action on fall
Belayer	0-3	0-0.9		Static belay
Climber’s footpath	-19	-5.7		Final impact position

Table 2-1: Heights of features on Air Guitar.



Figue 2-1 (right): Heights of features on Air Guitar.

Figure 2-2 (left): Sequence of events: 1) Climber falls and pulls out first protection; 2) rope snags belayer's arm creating static belay; 3) carabiner of second protection fails; 4) third protection is pulled out at almost the same time as climber strikes the belay shelf.



3. The preliminary investigation

The American Alpine Club (AAC) carried out a preliminary investigation of the accident [1]. This section summarizes the investigation report.

The investigation measured the accident site, and performed drop tests using the same equipment as was used during the accident. For drop testing, the #3 and #2 Camalots were used. The drop tests were statically belayed, and are described as follows in the preliminary investigation:

"For drop tests we used a nearly static belay on a 10.5 mm dynamic climbing rope. The belay was a loosely configured clove hitch on a large diameter carabiner, fixed to a large rock at the base of the climb. Simulating an "arm wrap" belay proved problematic. Using a static belay would create a "worst case" scenario on the Camalots, carabiners, and quick-draws. [The climber's] climbing rope was used for testing. Because of this, it should be noted that the elastic nature of the rope was compromised to some degree with each successive test."

The report summarizes the drop tests as follows:

"In summary, we found that a #3 Camalot properly placed in the vicinity of where [the climber] likely placed his protection could hold a 100 Kilo fall, even on static belay. We also found that a #3 Camalot improperly placed in the same area would not hold a 100 Kilo fall. The placement of the #3 Camalot is considered critical, as the forces applied to it during the fall were small compared to those applied to the #2 Camalot some 10-15 feet below.

We also found that a well placed #2 Camalot has the ability to hold a 100 Kilo fall on a static belay from ten and thirteen foot drops. A similar drop at fifteen feet, however, pulled the protection out of the crack."

The preliminary investigation quotes a report by J. Lee Davis, who studied the broken carabiner at the REI Laboratory. Davis' examination included hardness tests, electron microscope photography, and pull test of identical equipment. Davis' report is summarized in the preliminary investigation. The pull test is described as follows:

"A slow pull lab test using an identical #2 Camalot with identical quick-draw was conducted. The Camalot was set between two serrated steel parallel plates and the quick-draw was clipped into the wire loop of the Camalot. The bent gate carabiner of the quick-draw (opposite from the Camalot) was fixed to the base of the tensile test machine using a 12 mm greased pin. The gate of the bent gate carabiner was held open by a rubber band. The bent gate carabiner supported a maximum load of 8.67 kN before failure."

Davis reaches the following conclusions:

"All indications lead us to believe that the carabiner was manufactured correctly. [...] Evidence suggests that the wire-gate was open when the carabiner broke. If the wire-gate were closed there would likely be damage where it closes against the nose of the carabiner or

at the hinge of the gate. The broken carabiner had no such scaring or damage. Carabiners are designed to be strongest when closed and pulled along the spine. Having the gate open would increase the likelihood of the carabiner breaking when pulled at high forces."

The result of the preliminary investigation was reported in the magazine Rock and Ice [5]:

"Pro Pulled, Air Guitar (5.10a), Frenchman Coulee, Washington

Source: Mike Gauthier, edited by Jed Williamson

On September 30, 2002, the [climber] died from a fall while rock climbing. He was leading Air Guitar, a 65-foot 5.10a crack that requires precise nut and cam placements. [The climber] was near the top of the route when he fell some 60 feet to a rock ledge. Though wearing a helmet, he sustained fatal head injuries.

During the morning and early afternoon that day, [the climber] and his partner took turns leading sport routes. After climbing four or five bolted aretes, [the climber] took advantage of an opportunity to toprope a crack, Pony Keg (5.10a). Although [the climber] looked solid in the crack he told his partner that he found the climb challenging. [The climber] then decided to lead Air Guitar.

[The climber] started up the route, placing, in order, a small nut, two micro cams, and three small to medium cams. He fell near the top of the climb, the crux, shortly after placing a three-inch cam. That cam pulled, and the wire-gate carabiner clipped to the rope on the next cam broke, causing [the climber] to fall to the ledge.

Analysis:

This accident resulted from a series of combined incidents. [The climber] was relatively inexperienced at placing natural gear and, though a powerful athlete, was at his lead limit. The fact that the top cam pulled indicates that it was either placed incorrectly or walked to an insecure position, which is possible since he clipped all of his protection with short, stiff quickdraws. Another scenario is that [the climber] dislodged the piece by himself by kicking it with his foot as he climbed past it. Regardless, experienced natural-gear leaders are able to get solid protection at or near the same place [the climber's] cam pulled.

Subsequent studies of the broken carabiner revealed that the wire gate was not distressed; in other words the carabiner appears to have failed because its gate was open. While a gate-closed carabiner failure is rare, carabiners with their gates open lose as much as two-thirds of their strength, making failure in a fall a real possibility.

What caused the gate to open? It could have become wedged or constricted inside the crack because its short quickdraw would not let it lie outside the crack. Jammed in the crack, the carabiner could have had its gate pinned open. The short, stiff quickdraw could also have let the carabiner rotate into a cross-loading orientation, another extremely weak orientation.

Leading Air Guitar pushed [the climber's] crack-climbing abilities that day. Air Guitar and other 5.10a basalt column cracks like it are steep and require technical crack-climbing skills.

Mastering good crack-climbing skills takes extensive practice and training, which [the climber] did not have.

Air Guitar also requires the precise placement of natural protection. Learning how to properly size and place rock protection before attempting routes with hazardous fall exposure is important. Short quickdraws are best suited for sport climbing. When using natural protection, many climbers prefer slightly longer and more flexible quickdraws or slings, which provide for a smoother rope movement and decrease the chance of protection being displaced.

[Sidebar]

Safety Tips

Get in the habit of placing two pieces of protection just below the crux moves, and anywhere your protection is suspect. Doubling up also gives you an extra measure of safety in the event one piece fails in a fall. Also, when you place gear in a crack, be sure its quickdraw or sling is long enough to let the rope-end track outside of the crack. This will keep the carabiner from wedging in the crack, and having its strength compromised."

4. Additional information

Besides the preliminary investigation report, we have taken additional information into account, including descriptions by the belayer and other witnesses. We quote relevant excerpts below.

4.1 Witness reports in the Cascade Climbers discussion forum

The belayer published a detailed description of the accident in the Cascade Climbers internet discussion forum [6]. This description, as well as subsequent messages by other forum participants, provides many essential pieces of information.

4.1.1 Dynamic belay for first piece of protection, subsequently static

Soon after the accident, the belayer wrote:

"Just before I looked down to my feet while belaying, I saw [the climber] near the top, with a piece of protection by his foot. He had to have been about 20 meters up on the climb. We were using a 60-meter rope and earlier in the day, had plenty of extra rope when we rappelled from the anchors of the climbs that we were doing. Then I heard a commotion above me. [The climber] was falling.

He was falling and I saw his first piece pull. His rope went slack. My instinct was to duck and I crouched low into the corner to take up the slack. I think I pulled some rope through the belay device, but I am not sure. I did throw my left arm into the lead line to press it closer to the ground as I did crouch. It wrapped my arm once, caught my left biceps and cinched it. I was not wearing a shirt. It appeared after the fact that the belay action was delivered by the one loop around my arm that resulted in a full circle rope burn with trauma and I did not feel much pull on my belay device."

4.1.2 Belayer was unanchored

The belayer acknowledged he was belaying unanchored.

4.1.3 Climber used quickdraws

The belayer confirmed that the quickdraws used were Camp wire gate carabiners. These carabiners are specified to have closed gate strength 25kN, open gate strength 10kN, and minor axis strength 8kN.

4.1.3 Descriptions of the route Air Guitar

The signature "MichaelB" wrote:

"Over the past ten years, I have climbed Air Guitar more than 200+ times. On Sat I returned to Sunshine Wall to climb Air Guitar with the intention of more closely evaluating gear placements on the route. I certainly would not attempt to evaluate the event that led to [the climber's] death, but I do consider myself qualified to evaluate this particular route.

I do not believe the rock moved. Yes basalt columns do move over time, but the column that contains Air Guitar is well seated. The column is big, heavy, and the crack is reasonably solid for Vantage rock.

As I already knew, the route does take cams very well. However, as seasoned basalt, crack climbers know, basalt cracks are rarely perfectly parallel, and contain numerous irregularities. Consquently, a cam carefully placed in a concave surface is very bomber, while a hastily place cam that even partially contacts a convex surface can and often does walk.

Air Guitar and the nearby route, Pony Keg, both constrict as one reaches deeper into the crack. Ten feet below the Air Guitar anchor, one encounters a horizontal ledge. The hand crack just below the ledge will nicely take a carefully placed #3 camalot, or just one foot lower a more easily placed #2 camalot. The fist crack above the ledge likewise takes a well placed #4 camalot, however a #3 camalot placed in the fist crack will most likely pull because the crack does significantly flare at this point. Some climbers do find this last section to be a little tricky.

Five pieces of well placed gear is reasonably run-out for a basalt crack of this length. The climber would face potential twenty foot falls, which should be considered a risky consequence on basalt. I normally place six pieces, even though I have the route "wired". Due to the recent event, I placed eight pieces, which still involved a 25 foot runout to the first piece."

The signature "Kevin" wrote:

"While I generally agree that slings are better than qd's for trad, air guitar is so straight that no quick draw at all would probably have been fine. Air guitar is a clean crack in good rock and I would like to know what happened so I don't end up with a similar fate."

4.1.4 Observations of failed protection

John Crock, President of the Frenchman Coulee Climber Coalition, noticed the damage on the protection after the accident, but before it was used in testing during the preliminary investigation:

"I had a chance to examine the camalots and Metolius TCU that all pulled. One thing that [the belayer] has not made clear is that all pieces with the possible exception of the #2 camalot with the broken biner are quite visibly damaged. The #3 has a portion of the metal on the cams ground off by the shear force of ripping it from the rock. It was set at around 1/2 cam retraction. The #1 has a mm or two of metal removed by the rock on it's cams, the cams were set at nearly full retraction. The #3 TCU is dented and has a bent axle. The #1 camalot was pulled nearly to cable failure. I think both the #3 and #1 camalots have bent axles. They were heavily loaded and pulled out. The wire on the #2 camalot (note-all camalots in use were the older, two-stem wire type) visibly dented the carabiner it was clipped into. Bottom line- there was a lot of force on the cams. [The belayer] did not believe there was any rockfall associated with the fall."

4.2 Medical observations of belayer's arm

Adam Berkowicz, physician and forensic pathologist, and Jan Leyon, physician, states the following after examining a photograph of the belayer's arm [7]:

"We have examined a photograph of [the belayer's] arm, taken a few days after the accident. This shows two wounds circled around the upper left arm. Both wounds exhibit central ischemic paleness with excoriations on the sides. Distal to these finding s there is a widespread area of bleeding in the form of ecchymoses that covers a large area of the upper and lower left arm. The appearance of the above-mentioned injuries suggests a strangulation component of a large force, in order to achieve these injuries in the muscular arm. The injuries inflicted in [the belayer] are in concordance with a strangulation of the active rope around his left upper arm. We feel that there could hardly have been any slippage of rope."

Figure 4-1: Belayer's arm damage.

5. Analysis

In this section, we analyze the accident, taking into account the preliminary investigation [Section 3], witness reports [Section 4], a theoretical analysis of the relations between fall parameters [Appendix A], and computer simulations [Appendix B].

5.1 Estimate of fall parameters based on theoretical analysis

Our calculations are based on the assumption that the rope is linearly elastic. For very heavy falls (i.e. fall factors near 2), ropes generally do become non-linear, but for this accident, the fall factors were at most 1.1.

5.1.1 Rope elasticity indicated by the drop test

The investigators performing the preliminary investigation placed the #2 Camalot carefully in a well-set position at 12.9 m height (43 feet). Reportedly, two experienced climbers made the placement on top rope in cooperation. The test was second in the series of drop tests, so the elasticity of the rope should approximate that of the accident well.

The fall factor was 0.52. The #2 Camalot was of an older, dual-stem type. These are specified to have a breaking strength of 15 kN when tested according to the EN-standard [8], which specifies climbing conditions similar to those on Air Guitar. Assuming that a well-placed Camalot must be loaded to its breaking strength in order to fail, and assuming a friction coefficient of 0.3 for the carabiner-rope contact, the elasticity of the rope in terms of static elongation for a linearly elastic rope can be computed to have been at most 2.2 % [Appendix A, eq. 3-5].

An upper bound on the load, given the elasticity in terms of static elongation, as long as the climber is above the belayer, is shown in Diagram 5-1. The diagram is based on Eq. 3-9 in Appendix A.

5.1.2 Rope elasticity indicated by the #1 red Camalot

When the #1 Camalot was pulled out, 1-2 mm were ground off the cams [4.1.4]. It had been set at almost full retraction. This indicates that the #1 Camalot was overloaded. This Camalot is also of the older, dual-stem type, specified for 15 kN load.

The precise location of the #1 Camalot is unknown, but if we assume that the two pieces of protection between the #2 Camalot and the #2 yellow TCU were spaced evenly, the Camalot was set on approximately 10.4 m, giving a fall factor of 0.80. In order to create a pullout force of 15 kN, the elasticity corresponded to a static elongation of at most 2.7 % [Appendix A, eq. 3-5]. The Camalot was placed quite low, and simulation shows that it must have been pulled out just about the time of impact.

5.1.3 Rope elasticity indicated by the #3 red TCU

The #3 Metolius TCU is specified to have a breaking strength of 13.3 kN. It was pulled out, and the axle was bent, showing the TCU was overloaded.

The exact position of the #3 TCU is unknown, but again assuming approximately equal spacing, the Camalot would have been set on approximately 7.8 m, giving a fall factor of 1.1.

We can conclude it was pulled out *after* the first impact. The elasticity corresponded to a static elongation of at most 5.0 % [Appendix A, eq. 3-5]. It is noteworthy that the energy loss and speed due to the first impact did not prevent the TCU from being pulled out.

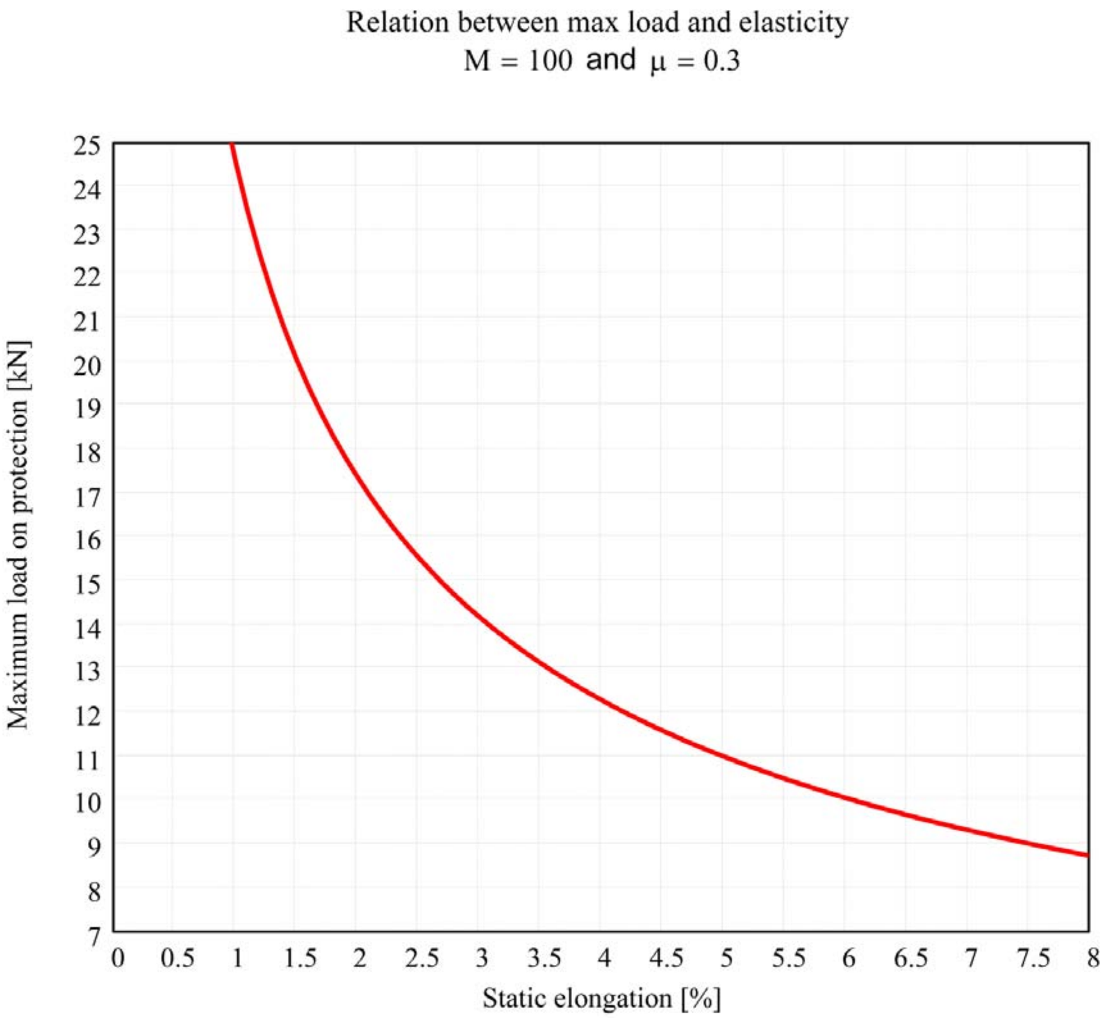


Diagram 5-1: Maximal load on protection as a function of rope elasticity. The climber is assumed to be above the belayer, and not to weigh more than 100 kg.

5.1.4 Load on #3 blue Camalot

When the topmost Camalot (blue #3) was pulled out, belay was still dynamic, according to the belayer's description. The fall factor was at most 0.17. The force on protection was limited by the belay device to approximately 4.9 kN, since this kind of belay device (Petzl Reverso) typically lets the rope slip through at a load of 2 kN [11].

5.1.5 Load on #2 yellow Camalot

The #2 Camalot was set at 12.3-13.8 m, giving a fall factor range of 0.41-0.59. With an elasticity corresponding to a static elongation of 1.8%, the force on the #2 Camalot could have been up to 17.0 kN [Appendix A, eq. 3-6]. This would have been enough to break the carabiner if the gate was open.

Had the static elongation been 6%, the force on the Camalot would have been less than 10.5 kN [Appendix A, eq. 3-6]. This is about the force required to break the carabiner with open gate. Had belay been dynamic, the force would have been limited to about 4.9 kN. This force would probably not have been enough to break the carabiner, even if the gate was open. The breaking strength of the carabiner with closed gate is specified as 25 kN. In order to exceed this limit, the static elongation of the rope would need to be less than 0.77 % [Appendix A, eq. 3-5].

5.1.6 Load on #1 red Camalot for a dynamic rope

If the rope had been dynamic with 6% static elongation, and both the #3 blue Camalot and the #2 yellow Camalot had failed, the force on the #1 Camalot would not have been larger than 11.4 kN [Appendix A, eq. 3-6]. This force would probably not have been enough to pull out the Camalot, but such an elastic rope would not have prevented a ground fall, anyway.

5.2 Estimate of fall parameters by computer simulation

We have run computer simulations of the fall dynamics, taking into account friction, as well as rope elasticity. Data from the simulations are presented below. We simulated a 100 kg-climber falling from a height of 17.4 m without slack. For the drop test, protection was placed at the known 12.9 m height, but for the other cases involving the #2 Camalot, protection was set at 12.3 m in order to obtain an upper bound on loads. We show the simulation results for ropes with 1.8% and 6% static elongation. The simulation program is given in Appendix B.

5.2.1 Simulation of the drop test

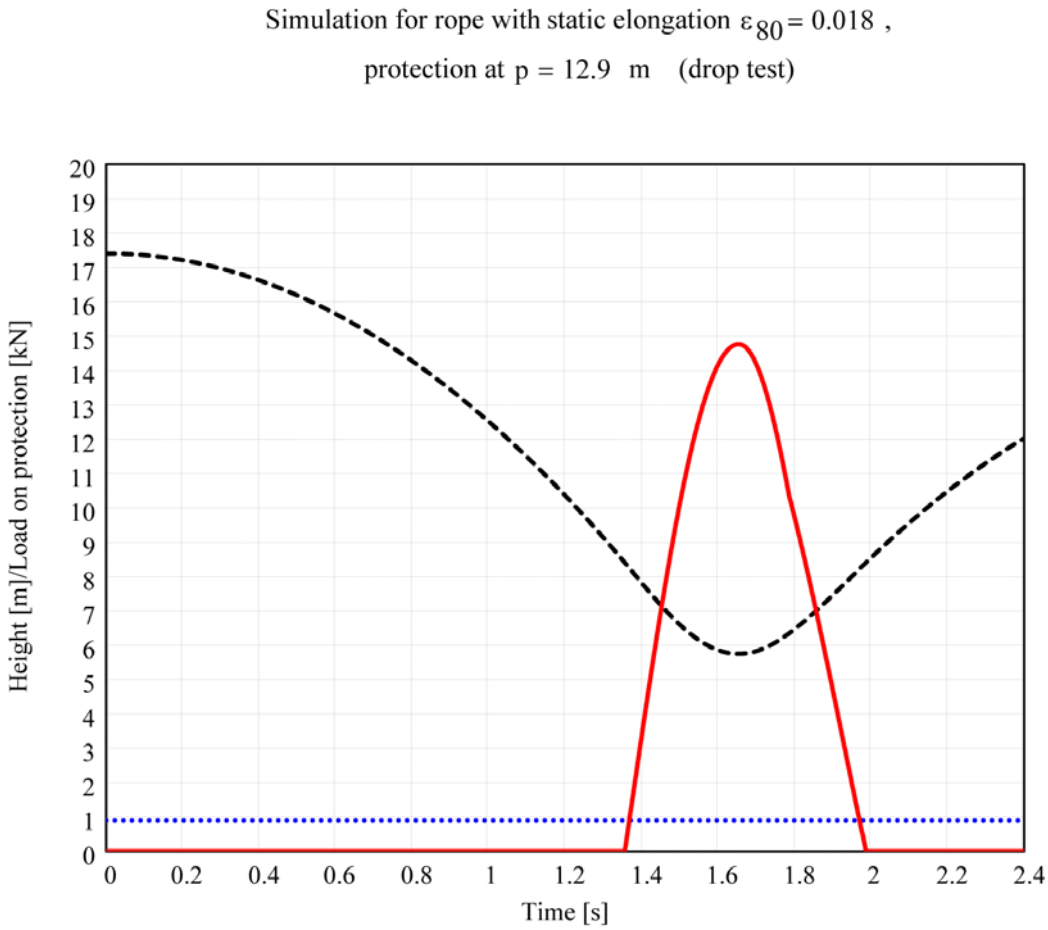


Diagram 5-2: Drop test simulation. The continuous line shows the load on protection in kilonewton [kN]. The dashed line shows the height over ground of the climber in meter [m]. The dotted line is the approximate height over ground of the "belayer", which was a large rock for the drop test.

5.2.2 Simulation for fall on static rope

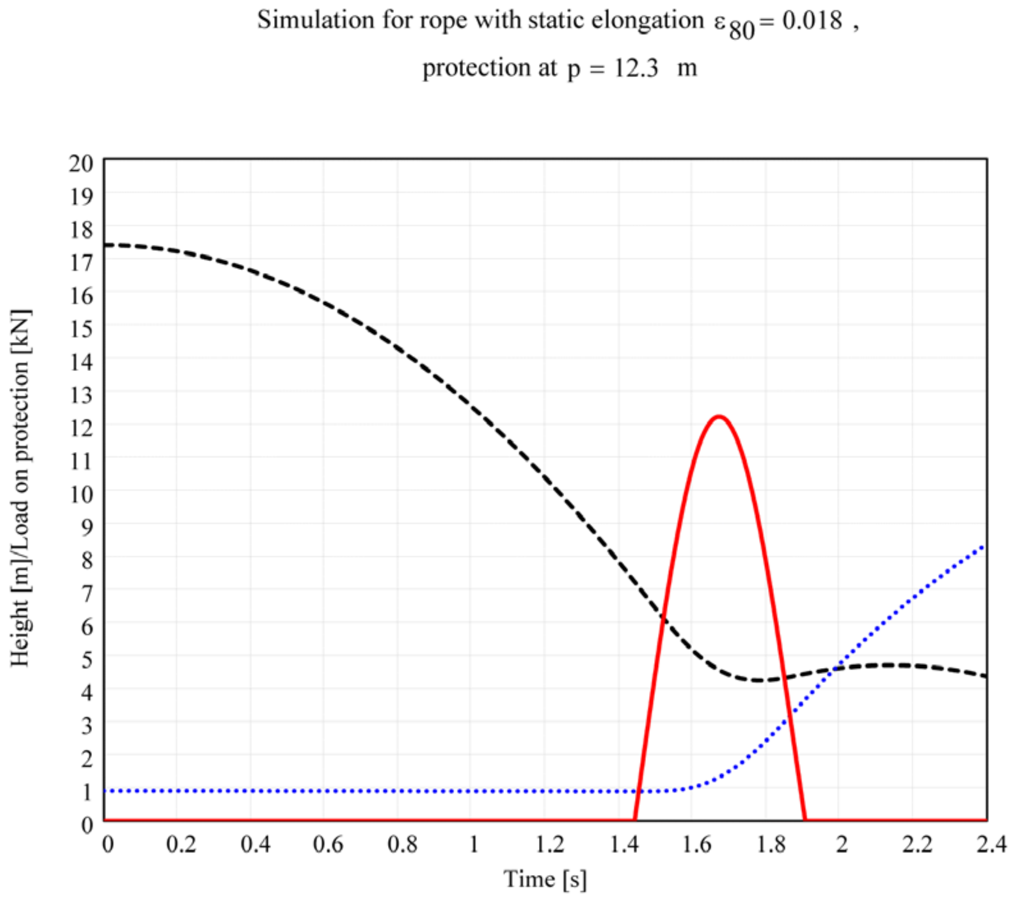


Diagram 5-3: Simulation for 1.8% static elongation. The peak load on protection is smaller here than in the drop test, since the belayer is unanchored and is hurled up in the air.

5.2.3 Simulation for fall on dynamic rope

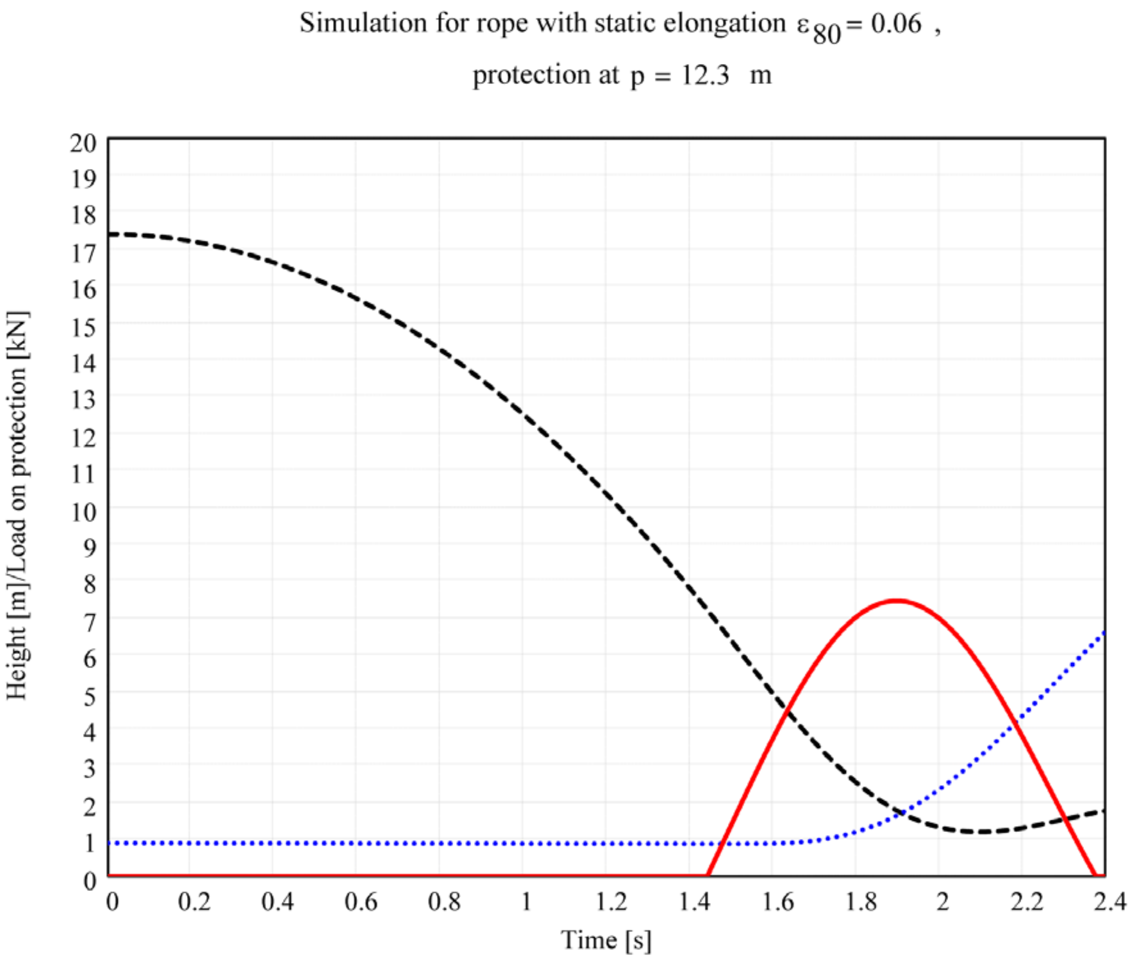


Diagram 5-4: Simulation for 6% static elongation. Comparing this diagram with the one for 1.8% static elongation shows that the load is now more spread out and the peak force is lower. However, the climber is closer to the ground and nearly touches down.

6. Discussion

6.1 Possible causes

6.1.1 Dubious topmost piece of protection

The fall factor was small (less than 0.17), and the belay at this point was still dynamic. The load on the #3 blue Camalot accordingly appears to have been less than 4.9 kN. Even if the rope was static, the difference between a dynamic rope and a static rope is minor for such small fall factors. Reportedly, it is difficult to set a #3 Camalot on this height, since the device is somewhat small for this position, and the crack is *flared*, i.e. expands outwards. The most likely explanation for the first piece of protection pulling out seems to be it wasn't perfectly set.

6.1.2 Static belay

At the second piece of protection, the belay became static, as can be understood from the belayer's description [4.1.1]. The absence of rope slip is confirmed by a forensic analysis of the damage on the belayer's arm [4.2].

If belay had been dynamic, the load on the protection would have been limited to about 4.9 kN, and the protection would probably have held, although the belayer may have burnt his braking hand on the rope when pulled through the belay device.

6.1.3 Dynamic rope turned static

The strongest argument for an inelastic rope is the preliminary investigation's drop test 2. The fall factor was approximately 0.52, which is small, but the shock load was still large enough to pull out a well-placed #2 Camalot, which means that the shock load would have been on the order of 15 kN. As a comparison, for a new dynamic rope, the peak load on a falling 80-kg climber for the standardized fall [13] with the large fall factor 1.77, is typically about 8 kN, or about 15 kN on the protection. This shows that much of the elasticity of the rope was lost, without assuming linear elasticity.

If we do assume linear elasticity, the drop test shows that elasticity of the rope corresponds to a static elongation of at most 2.2% (theoretical analysis) and at most 1.8% (computer simulation). Also, the third and fourth pieces of protection were apparently overloaded and pulled out during the accident. We have collected these events, together with the consistent maximum static elongation in Table 6-1, using both theoretical analysis and computer simulation.

These data suggest that the originally dynamic rope had lost elasticity,and become more static. It seems that reasonably intensive rappelling and top roping preceded the accident, which certainly reduced the elasticity, but it is unclear if this alone explains the reduction in elasticity. The preliminary investigation notes that *"…[The belayer and climber] had successfully belayed each other on four (possibly five) routes before the accident occurred"*, implying that they had climbed 8-10 pitches in approximately 4 hours. In this report, we cannot answer the complex question of exactly why the rope had lost elasticity.

6.1.4 Carabiner open on fall

An open gate is consistent with the laboratory analysis of the broken carabiner [Section 3]. With a closed gate, the carabiner was specified to hold 25 kN. It seems unlikely such high forces on the protection could be created during the accident, since this would imply an extremely inelastic rope. This supports the open-gate theory. The preliminary investigation suggests that the carabiner could have been constricted in the crack, and the gate kept open, due to the stiffness of the quick-draw.

Event	Consistent static elongation (theory)	Consistent static elongation (simulation)
Drop test: Pulled-out #2 Camalot, 15 kN at 12.9 m	<2.2%	<1.8%
Broken carabiner 8.67 kN at >12.3 m	(no constraint)	<4.1%
Pulled-out #1 Camalot 15kN at 10.4 m	<2.7%	<1.5%
Pulled-out #3 TCU 13.3 kN at 7.8 m	<5.0%	<2.4 %

Table 6-1: Evidence supporting an inelastic rope theory.

6.1.5 Other factors

The average distance between the top four pieces of protection was 3 m (10 feet), which is not out of the ordinary. The outcome of the accident was indeed influenced by an apparently unsatisfactory placement of the topmost piece of protection, but does not seem to be affected significantly by the placements of the other pieces of protection. The failures of the third and fourth pieces of protection from the top appear to be due to the combination of a reduced-elasticity rope and static belay.

6.2 What if…?

Belay	Rope	Gate	Max theor. load [kN]	Simul. load [kN]	Strength [kN]	Result
Static	1.8%	Open	17.0	12.2	8.67	Prot. failure
Static	1.8%	Closed	17.0	12.2	15	OK
Static	6%	Open	10.5	7.5	8.67	OK
Static	6%	Closed	10.5	7.5	15	OK
Dynamic	1.8%	Open	4.9		8.67	OK
Dynamic	1.8%	Closed	4.9		15	OK
Dynamic	6%	Open	4.9		8.67	OK
Dynamic	6%	Closed	4.9		15	OK

Table 6-2: Probable results for different combinations of conditions.

In Table 6-2, we summarize what most likely would have happened at the second piece of protection from the top, depending on the three factors belay, rope elasticity in terms of static elongation, and the state of the carabiner gate.

6.3 Alternative theories

We have four potential causes of the accident:

A) The first piece of protection was not well set.

B) The carabiner was open on fall.

C) Belay was static.

D) The rope was inelastic.

Although it seems these factors were all present, not all of them must necessarily have affected the outcome of the accident. We can distinguish four major alternatives: ABCD, ABC, AB, and ACD.

6.3.1 Alternative theory ABCD: Open-gate carabiner failed due to static belay and inelastic rope

The rope was apparently sufficiently inelastic to break the carabiner with open gate. This alternative makes no assumption about about the carabiner being additionally constrained, and appears to be the most likely explanation.

6.3.2 Alternative theory ABC: Open-gate carabiner would have failed even for a fully dynamic rope

It is known that constricted carabiners sometimes fail for stresses, possibly lower than the peak load created by a dynamic rope. This could have happened, but it appears unlikely. As far as we know, there is no evidence of such constricted failure. Since this alternative makes the additional assumption of constricted placement, it is less likely than the ABCD theory.

6.3.3 Alternative theory AB: Open-gate carabiner would have failed even for dynamic belay

In exceptional situations, carabiners may fail for loads that are lower even than those created when using dynamic belay. If this were the case, the climber would have made a ground fall despite being dynamically belayed. It is also a possible alternative, but more unlikely than the ABC alternative.

6.3.4 Alternative theory ACD: Protection would have failed even with closed carabiner

Since belay was static and the rope inelastic, the Camalot #2 might have pulled out if the carabiner gate had been closed. During fall testing, the preliminary investigation found that placing the Camalot slightly higher than it was set for drop test #2, the Camalot could hold a fall. Since this alternative makes an assumption on the strength of the Camalot #2 placement, it is less likely than the ABCD theory.

6.4 Additional measurements

This report is based on a thorough analysis of all data available to us at the time of writing. Although we believe the material suffices for reaching a conclusion, additional investigations and experiments are possible.

6.4.1 Identification of rope

Identifying the rope, its manufacturer, age, and history, would be useful in order to solve the difficult question of why the rope lost elasticity.

6.4.2 Analysis of the failed protection below the #2 Camalot

Detailed analysis of the failed protection below the #2 Camalot (#1 Camalot and #3 TCU) may provide an opportunity to accurately estimate the pullout load, from which the rope elasticity can be computed. As far as we understand, these were not used in the drop tests.

6.4.3 Rope-carabiner friction coefficient

Currently, we approximate the friction coefficient between rope and carabiner by a standard value of 0.3. More accurate values would allow more precise calculations, although the analysis in the appendix shows that the bounds on load change only slightly with a change in friction.

6.4.4 Measurement of rope elasticity

The elasticity of the rope was unfortunately not measured during the initial investigation. During testing, the elasticity of the rope was probably further reduced. It can also happen that a rope regains some elasticity after a recovery period. For this reason, measuring the rope in its current state may not produce completely reliable data. Nevertheless, the elasticity is not likely to be radically different, and testing may confirm the rope's elasticity properties. The rope's current strain-stress properties have been measured at the department of Solid Mechanics, KTH (Royal Institute of Technology, Stockholm), where the equipment is being analyzed.

It should be noted that only a 47-m section of the originally 60-m long rope was used for testing during the preliminary investigation. Some unknown person removed part of the rope from the site before the remainder of the equipment was collected. Since this piece of rope was unaffected by fall testing, it may more accurately display the original elasticity properties of the rope.

7. Conclusions

Apparently, belay was static from the second piece of protection from the top. There is evidence that the rope had reduced elasticity, and that a critical carabiner was open. It is likely that the first protection was too small and/or not well set. Most likely, these four factors together caused the accident. Our calculations are mainly based on the assumption that the rope is approximately linearly elastic for fall factors less than one, but one of the drop tests show this isn't necessary for concluding that the rope was inelastic.

One cause of this accident we find particularly disturbing. Despite both the belayer and climber being experienced, the static nature of the rope went unnoticed. This indicates that dynamic ropes losing elasticity may be underestimated climbing hazards. As far as we know, there are currently no good methods for supervising the elastic status of climbing ropes. The accident shows that this problem may deserve further investigation, and that procedures for monitoring rope elasticity may need to be developed.

We second the safety tips as published in the Rock and Ice magazine [5]. Additionally, we would like to suggest that:

a) Climbers and belayers need to be aware of the elasticity of the rope, particularly when lead climbing after intensive rappelling and top-roping.
b) Belayers should take care not to touch the live part of the rope on a fall, in order not to accidentally create a static belay.

We would also like to mention the following secondary suggestions:

c) Climbers are recommended to use locking carabiners for critical protection;
d) Belayers are recommended to be anchored when belaying lead climbers;
e) Belayers are recommended to use braking gloves when there is risk for heavy falls.

Incidentally, if a similar accident does happen, analysis would be simplified if the locations of all remaining equipment are documented before removal; equipment is documented before use, e.g. in drop testing; the rope elasticity is investigated if there are multiple protection failures; and the elasticity of the rope is measured before drop testing.

8. Acknowledgements

It would not have been possible to carry out this investigation had it not been for the belayer's commendable initiative to discuss the accident on the Cascade Climbers Internet forum. The forum provided important information from many experienced climbers. The preliminary investigators made great efforts to perform drop testing, measurements, and a review of the REI laboratory report, on which we could base our analysis. Also, we are grateful to Black Diamond for providing data on the Camalot camming devices; to Dr. Adam Berkowicz for the forensic analysis; and to Dr. Gunnar Sjödin for proof reading.

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Appendix A: The physics of a fall

Relations between fall factor, rope elasticity, and load on protection

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Abstract: *This appendix describes how the elasticity of a climbing rope can be estimated, given the load on protection during a fall. We derive equations relating the fall factor, elasticity, and load on protection for conditions including friction and energy dissipation during the fall.*

1. Introduction

The *fall factor*, f [unit m/m], is traditionally defined as the distance d [m] a climber falls before the rope starts to strain, divided by the amount of rope L [m] between the belayer and the climber, $f = d / L$. We shall use an auxiliary entity r [m/m] for *rope-relative fall distance*, defined by $r = f + \varepsilon$, where ε [m/m] is the *rope strain*. r will vary during a fall. It can also be described as the current fall distance divided by the rope length. For most common climbing situations, $0 \le r \le 1$. $r > 1$ means that the climber has passed the belayer on the way down, normally an undesirable situation.

There are five questions of primary interest:

- *What is the rope elasticity, given the fall factor and load on the protection?*
- *What is the rope elasticity, given the load on the protection (for all fall factors)?*
- *What is the load on protection, given the fall factor and elasticity?*
- *What is the load on protection, given the elasticity (for all fall factors)?*
- *What is the fall factor, given the elasticity and load on protection?*

Below, we derive formulas for these relations. First we assume that the rope is linearly elastic, and that there is no friction between rope and protection, and following that, we derive the formulas for the case of friction between rope and protection.

2.1 Linearly elastic rope, no friction in top carabiner

In the following, entities with index x refers to the section of rope between the climber and the topmost piece of protection, and index y refers to the section of rope between the belayer and the protection. L is the length of the rope between the belayer and climber, and L_x/L and L_y/L are the (mass) fractions of rope between protection and climber, and protection and belayer, respectively.

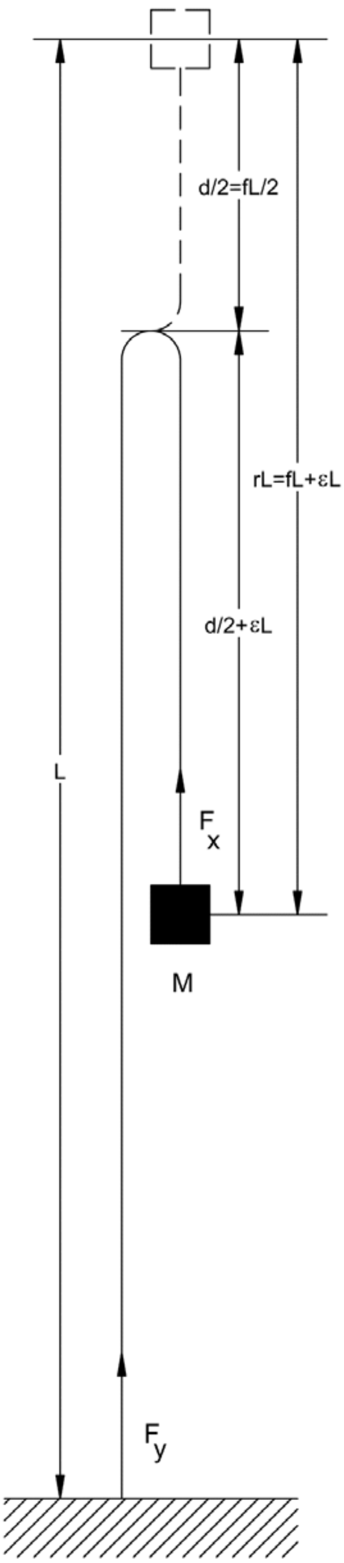


Figure A-1: Notation

We will first find an expression for r in terms of load on the protection. For a linearly elastic rope we have $F_x = F_y = k \cdot \Delta L$, where k is the *spring constant* [N/m] and $\Delta L = \Delta L_x + \Delta L_y$ is the *absolute rope elongation* [m]. We can express the formula in terms of strain by introducing the rope *stiffness* $c = kL$ [N] and writing $F_x = F_f = c \cdot \varepsilon$. c is a material constant for the rope and is independent on its length. It can be computed from the *static elongation* ε_{80} [m/m], which is a standard way of characterizing the elasticity. The static elongation is the strain of the rope for a static load of 80 kg, so $c = 80g/\varepsilon_{80}$, where g is the *ground acceleration*, $g = 9.80665\,\text{m/s}^2$.

The load on the protection is $F = F_x + F_y = 2F_x$ [N] so we have

$$r = f + \varepsilon = f + F_x/c = f + F/2c. \qquad (2\text{-}1)$$

Now we shall derive a second equation relating F, r, and c based on an energy argument. When the climber falls, the lost potential energy is converted into other types of energy: *kinetic energy E_{kin}* of the falling climber and the rope, *potential energy E_{rope}* in the rope, and *dissipated energy E_{diss}*, e.g. friction heat in the rope, knots, protection and carabiners, as well as turbulence, vibrations in the climber and rock, etc.,

$$E_{pot} = E_{rope} + E_{kin} + E_{diss} \geq E_{rope}.$$

Here,

$$E_{pot} = MgrL,$$

where M is the mass of the falling climber. We have

$$E_{rope} = \int_0^{\Delta L} F_x ds = \int_0^{\Delta L} k \cdot s\, ds = \frac{k \cdot \Delta L^2}{2} = \frac{c\varepsilon^2 L}{2} = \frac{LF_x^2}{2c}.$$

$E_{pot} \geq E_{rope}$ implies that

$$Mg \cdot rL \geq \frac{LF_x^2}{2c} = \frac{LF^2}{8c}$$

i.e.

$$F^2 \le G_0 \cdot rc \text{, where } G_0 = 8Mg \quad . \tag{2-2}$$

2.2 What is the elasticity?

We are looking for c expressed in terms of f and F. Substituting (2-1) into (2-2) produces

$$F^2 \le G_0 \cdot rc = G_0(f + F/2c)\,c = G_0 fc + G_0 F/2 \,.$$

From this,

$$c \ge \frac{F^2 - G_0 F/2}{G_0 f} \,. \tag{2-3}$$

2.3 What is the load on protection?

From (2-3) we obtain

$$F^2 - G_0 F/2 + \frac{G_0{}^2}{16} \le G_0 fc + \frac{G_0{}^2}{16} \,,$$

implying

$$F \le \frac{G_0}{4}\left(1 + \sqrt{\frac{16fc}{G_0} + 1}\right). \tag{2-4}$$

2.4 What is the fall factor?

From (2-3),

$$f \ge \frac{F^2 - G_0 F/2}{G_0 c} \,. \tag{2-5}$$

For the common case $r \le 1$, i.e. before the climber reaches the belayer on the way down, we additionally have

$$f = r - \varepsilon \le 1 - \varepsilon = 1 - \frac{F}{2c} \,. \tag{2-6}$$

and from (2-2),

$$F^2 \le G_0 c \,. \tag{2-7}$$

3. Linearly elastic rope, friction

When there is friction in the top carabiner, we have $F_y = (1-\mu)F_x$, where μ [N/N] is the *friction coefficient*, typically with a value around 0.3, so

$$F = F_x + F_y = (2-\mu)F_x \qquad (3\text{-}1)$$

Strictly speaking, since the rope is wound the angle π around a cylindrical object (the carabiner), μ is not the traditional Coulomb friction coefficient μ_0 between the rope material and the carabiner material, but is numerically $\mu = \exp(\pi\mu_0)$.

Also,

$$\varepsilon = \frac{1}{L}(L_x\varepsilon_x + L_y\varepsilon_y) = \frac{1}{L}(L_x\frac{F_x}{c} + L_y\frac{F_y}{c}) = \frac{F_x}{c}(\frac{L_x}{L} + (1-\mu)\frac{L_y}{L}).$$

For falls without slack, i.e. from height L above the belayer, we observe that

$$r - \frac{f}{2} \geq \frac{L_x}{L} \geq \frac{f}{2}. \qquad (3\text{-}2)$$

Using this inequality,

$$\frac{L_x}{L} + (1-\mu)\frac{L_y}{L} \leq r - \frac{f}{2} + (1 - r + \frac{f}{2})(1-\mu) = 1 - \mu + \mu r - \mu\frac{f}{2},$$

so

$$r = f + \varepsilon \leq f + \frac{F_x}{c}(1 - \mu + \mu r - \mu\frac{f}{2})$$

from which

$$r \leq f + F\frac{1-\mu + f\mu/2}{2c - \mu F}. \qquad (3\text{-}3)$$

The elastic energy stored in the rope is

$$E_{rope} = \frac{L_x F_x^2}{2c} + \frac{L_y F_y^2}{2c} = \frac{LF_x^2}{2c}(\frac{L_x}{L} + \frac{L_y}{L}(1-\mu)^2).$$

Using inequality (3-2) again, we have

$$\frac{L_x}{L}+\frac{L_y}{L}(1-\mu)^2 \ge \frac{f}{2}+(1-\frac{f}{2})(1-\mu)^2 = (1-\mu)^2 + f(\mu-\mu^2/2)$$

so

$$E_{rope} = \frac{LF_x^2}{2c}(\frac{L_x}{L}+\frac{L_y}{L}(1-\mu)^2) \ge \frac{(1-\mu)^2 + f(\mu-\mu^2/2)}{(2-\mu)^2}\frac{LF^2}{2c}$$

and

$$Mg \cdot r \ge \frac{(1-\mu)^2 + f(\mu-\mu^2/2)}{(2-\mu)^2}\frac{F^2}{2c},$$

or, equivalently,

$$F^2 \le G_\mu \cdot rc \text{ where } G_\mu = \frac{(1-\mu/2)^2}{(1-\mu)^2 + f(\mu-\mu^2/2)} \cdot 8Mg\,. \qquad \text{(3-4)}$$

We know that the fall factor $f \le 2$. For all "normal" friction situations $0 \le \mu < 1$, so $\mu - \mu^2/2 \ge 0$. For small μ, we can approximate

$$G_\mu \le \left(\frac{1-\mu/2}{1-\mu}\right)^2 \cdot 8Mg \approx (1+\mu)\cdot 8Mg = (1+\mu)\cdot G_0\,. \qquad \text{(3-4b)}$$

This expression allows us to easily estimate the force, given μ, M, r, c when f is unknown.

3.2 What is the elasticity?

We are looking for c expressed as a function of f and F. Substituting (3-3) into (3-4) produces

$$F^2 \le G_\mu \frac{2cf + F(1-\mu-f\mu/2)}{2\mu c - \mu F}c\,.$$

A lower bound for c can be obtained from the quadratic equation

$$c^2 + c\frac{G_\mu F(1-\mu-\mu f/2) - 2F^2}{2G_\mu f} + \frac{\mu F^3}{2G_\mu f} \ge 0\,. \qquad \text{(3-5)}$$

3.3 What is the load on protection?

(3-5) is a cubic equation in F,

$$F^3 - F^2 \frac{2c}{\mu} + F \frac{G_\mu c}{\mu}(1 - \mu - \frac{\mu f}{2}) + \frac{2 G_\mu c^2 f}{\mu} \geq 0\,, \tag{3-6}$$

The relevant solution is the "middle" solution, which gives an upper bound on F.

3.4 What is the fall factor?

From (3-6),

$$\frac{F}{4} \frac{4Fc - 8Mgc(1-\mu) - \mu F^2}{Mgc\left[4c - F\mu\right]} \leq f \leq 2\,. \tag{3-7}$$

This equation gives us an upper bound on the load *for all fall factors* given the elasticity of the rope, the mass, and the friction, i.e. we don't need to know the distance fallen in order to compute the bound.

For the common case $r \leq 1$, we additionally have

$$\begin{aligned} 1 \geq r = f + \varepsilon = f + \frac{L_x}{L}\varepsilon_x + \frac{L_y}{L}\varepsilon_y = f + \frac{L_x}{L}\varepsilon_x + \frac{L_y}{L}(1-\mu)\varepsilon_x \geq \\ \geq f + \frac{f}{2}\varepsilon_x + (1 - \frac{f}{2})(1-\mu)\varepsilon_x = f(1 + \frac{\mu\varepsilon_x}{2}) + (1-\mu)\varepsilon_x\,. \end{aligned}$$

Extracting f,

$$f \leq \frac{1-(1-\mu)\varepsilon_x}{1+\varepsilon_x \mu/2} = \frac{(2-\mu)c - (1-\mu)F}{(2-\mu)c + \mu F/2}\,. \tag{3-8}$$

Equations (3-7) and (3-8) can be combined in order to find a relation between c and F not involving f,

$$\frac{F}{4} \frac{4Fc - 8Mgc(1-\mu) - \mu F^2}{Mgc\left[4c - F\mu\right]} \leq f \leq \frac{(2-\mu)c - (1-\mu)F}{(2-\mu)c + \mu F/2}\,. \tag{3-9}$$

Appendix B: Computer simulation

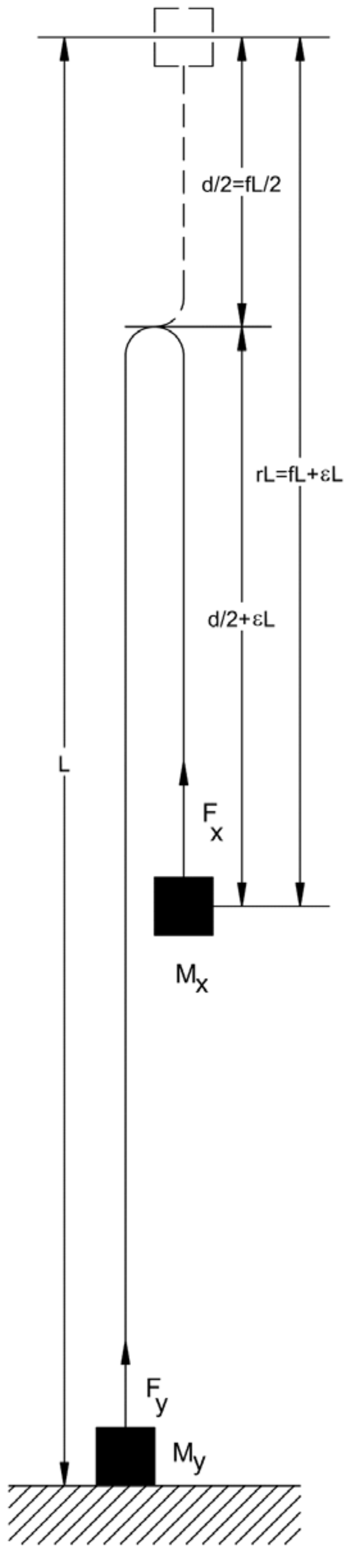
Numerical simulations allow more precise calculation of fall parameters than the theoretical analysis, since computation is possible without many simplifying approximations.

The computer simulations here produce tighter bounds than the previous theoretical analysis mainly for several reasons: The L_x/L approximation (eq. 3-2) becomes unnecessary; simulation takes energy loss due to friction into account; simulation also takes into account that the belayer was unanchored. (For the drop tests, however, the rope was anchored [1, p. 7], which can also easily be accounted for.) Another advantage of the simulations is that we can illustrate the positions and forces as they develop over time.

The simulation program and algorithm are expressed in the MathCAD language, a straightforward and easy-to-read programming language for mathematical computations [9].

In principle, the program solves a differential-algebraic equation using a forward-Euler scheme. Since the simulated time is short (2.4 s), there are no problems with numeric instability, as sometimes happens with forward-Euler. One advantage of using this method is that it can handle discontinuous derivatives well, such as generated by Coulomb friction.

We have used 2000 steps for the simulation. Reducing the step size doesn't significantly alter the results.



Dynamic simulation of falling climber

Martin Nilsson 2004-05-19

1. Physical properties

$g \equiv 9.80665$

$k_s(s_1, s_2) := k_s$

$$K_s(s_1, s_2, d) := \begin{cases} 0 & \text{if } |s_2 - s_1| \ge |d| \\ -\text{sign}(d)\cdot k_s(s_1, s_2)\cdot\left(\dfrac{|d|}{|s_2 - s_1|} - 1\right) & \text{otherwise} \end{cases}$$

$$\text{belayerBehaviour}(y, y_t) := \begin{cases} 0 & \text{if } y \ge y_0 \vee y_t \ge 0 \\ 2g & \text{otherwise} \end{cases}$$

2. Equilibrium equations

$F_x(x, x_t, s, s_t) := K_s(0, s, x - p)$

$F_y(y, y_t, s, s_t) := K_s(s, L, y - p)$

$$x_{tt}(x, x_t, y, y_t, s, s_t) := \frac{F_x(x, x_t, s, s_t)}{M_x} - g$$

For unanchored belayer:

$$y_{tt}(x, x_t, y, y_t, s, s_t) := \left(\frac{F_y(y, y_t, s, s_t)}{M_y} - g\right) + \text{belayerBehaviour}(y, y_t)$$

For drop test: $y_{tt}(x, x_t, y, y_t, s, s_t) := 0$

3. Friction equations

$\text{slipEq}(c, x, x_t, y, y_t, s, s_t) := c\cdot F_x(x, x_t, s, s_t) - F_y(y, y_t, s, s_t)$

$$\text{slipdirection}(F_x, F_y) := \begin{cases} 1 & \text{if } \left[(1 - \mu)\cdot F_x \ge F_y\right] \wedge F_x \ne 0 \wedge F_y \ne 0 \\ -1 & \text{if } \left[(1 - \mu)\cdot F_y \ge F_x\right] \wedge F_x \ne 0 \wedge F_y \ne 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{freefall}(x,y) := \begin{pmatrix} \dfrac{|p-x|\cdot L}{|p-x| + |p-y|} & 0 \end{pmatrix}$$

$$\mathrm{slip}(x, x_t, y, y_t, s, \mathrm{dir}) := \left| \begin{array}{l} s_{old} \leftarrow s \\ c \leftarrow (1-\mu)^{\mathrm{dir}} \\ s \leftarrow \mathrm{root}\left(\mathrm{slipEq}\left(c, x, x_t, y, y_t, s, \dfrac{s - s_{old}}{\Delta t}\right), s\right) \\ s \leftarrow s_{old} \ \text{ if } (\mathrm{dir} = -1 \wedge s_{old} < s) \vee (\mathrm{dir} = 1 \wedge s_{old} > s) \\ \begin{pmatrix} s & \dfrac{s - s_{old}}{\Delta t} \end{pmatrix} \end{array} \right.$$

4. Simulation

$$\mathrm{rope}(x, x_t, y, y_t, s, s_t) := \left| \begin{array}{l} F_y \leftarrow F_y(y, y_t, s, s_t) \\ F_x \leftarrow F_x(x, x_t, s, s_t) \\ \mathrm{dir} \leftarrow \mathrm{slipdirection}(F_x, F_y) \\ \mathrm{freefall}(x,y) \ \text{ if } F_x = 0 \vee F_y = 0 \\ (s \ \ 0) \ \text{ if } \mathrm{dir} = 0 \wedge F_x \neq 0 \wedge F_y \neq 0 \\ \mathrm{slip}(x, x_t, y, y_t, s, \mathrm{dir}) \ \text{ if } \mathrm{dir} \neq 0 \wedge (F_x \neq 0 \wedge F_y \neq 0) \end{array} \right.$$

$$\mathrm{next}(\mathrm{state}) := \left| \begin{array}{l} (x \ \ x_t \ \ y \ \ y_t \ \ s \ \ s_t) \leftarrow \mathrm{state}^T \\ X \leftarrow x + x_t \cdot \Delta t \\ X_t \leftarrow x_t + x_{tt}(x, x_t, y, y_t, s, s_t) \cdot \Delta t \\ Y \leftarrow y + y_t \cdot \Delta t \\ Y_t \leftarrow y_t + y_{tt}(x, x_t, y, y_t, s, s_t) \cdot \Delta t \\ (s \ \ s_t) \leftarrow \mathrm{rope}(X, X_t, Y, Y_t, s, s_t) \\ (X \ \ X_t \ \ Y \ \ Y_t \ \ s \ \ s_t)^T \end{array} \right.$$

$$\mathrm{simulate}(\mathrm{initialstate}) := \left| \begin{array}{l} \mathrm{state}^{\langle 0 \rangle} \leftarrow \mathrm{initialstate} \\ \text{for } i \in 0, 1 .. N-1 \\ \quad \mathrm{state}^{\langle i+1 \rangle} \leftarrow \mathrm{next}\left(\mathrm{state}^{\langle i \rangle}\right) \\ \mathrm{state} \end{array} \right.$$

5. Parameters

Climber mass [kg]:	$M_x \equiv 100$
Belayer mass [kg]:	$M_y \equiv 90$
Initial height of climber [m]:	$x_0 \equiv 17.4$
Initial speed of climber [m/s]:	$x_{t0} \equiv 0$
Height of protection [m]:	$p \equiv 12.9$
Initial height of belayer [m]:	$y_0 \equiv 0.9$
Initial speed of belayer [m/s]:	$y_{t0} \equiv 0$
Length of rope between climber and brake [m]:	$L \equiv \left\|x_0 - y_0\right\|$
Height of anchor [m]:	$b \equiv 0$
Friction coefficient protection-rope [1]:	$\mu \equiv 0.3$
Rope spring force function [N]:	$K_s(s_1, s_2, d)$

(Force to stretch rope section between s1 and s2 to length d.)

Rope static elongation for 80 kg load [1]:	$\varepsilon_{80} \equiv 0.018$
Rope spring constant (when linear) [N]:	$k_s \equiv 80 \dfrac{g}{\varepsilon_{80}}$
Belayer behaviour function [N]:	$\mathrm{belayerBehaviour}(y, y_t)$
Number of time steps [1]:	$N \equiv 2000$
Time step [s]:	$\Delta t \equiv \dfrac{2.4}{N}$

$$\mathrm{initialstate} := \left(x_0 \quad x_{t0} \quad y_0 \quad y_{t0} \quad \frac{L \left|x_0 - p\right|}{\left|x_0 - p\right| + \left|y_0 - p\right|} \quad 0 \right)^T$$

$t := 0, 1 .. N$

$U := \mathrm{simulate}(\mathrm{initialstate})$

$F_x(i) := F_x\left(U_{0,i}, U_{1,i}, U_{4,i}, U_{5,i}\right)$

$F_y(i) := F_y\left(U_{2,i}, U_{3,i}, U_{4,i}, U_{5,i}\right)$

$F_P(i) := F_x(i) + F_y(i)$

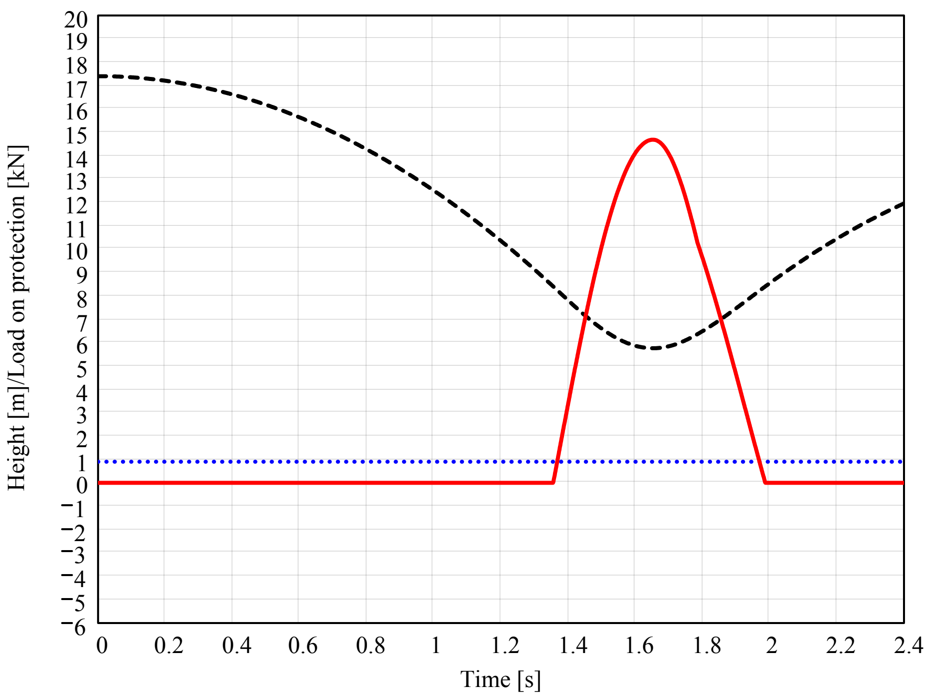
$x(t) := U_{0,t}$

$y(t) := U_{2,t}$

$s_t(t) := U_{5,t}$

Simulation for rope with static elongation $\varepsilon_{80} = 0.018$,

protection at $p = 12.9$ m



Dotted line = belayer
Dashed line = climber
Continuous line = load on protection